Yukawa coupling structure in intersecting D-brane models

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Abstract. The structure of Yukawa coupling matrices is investigated in type IIA $T^6/(Z_2 \times Z_2)$ orientifold models with intersecting D-branes. Yukawa coupling matrices are difficult to be made realistic in conventional models in which the generation structure emerges by the multiple intersection of D-branes in the factorized $T^6 = T^2 \times T^2 \times T^2$. We study the new type of flavor structure, where Yukawa couplings are dynamically generated, and show this type of models lead to non-trivial structures of Yukawa coupling matrices, which can be realistic.

1 Introduction

Understanding the masses and flavor mixing of quarks and leptons is one of the most important issues in particle physics. New physics which derives Yukawa coupling matrices in the standard model should certainly exist. Yukawa couplings, in a sense, seem naturally of $\mathcal{O}(1)$. Thus, how to derive suppressed Yukawa couplings is a key to understand the hierarchy of fermion masses and mixing angles.

String theory, which is a candidate for a consistent theory of quantum gravity, gives an attractive framework to generate Yukawa coupling matrices. Indeed, in several types of string models coupling selection rules have been studied and Yukawa couplings have been calculated. Recent development of the models with intersecting D-branes, intersecting D-brane models (see [1–4] for the essential idea), opens a new possibility towards realistic models. Among them, models with low-energy supersymmetry [4–15] are interesting, because those are constructed as stable solutions of the string theory. Within the framework of intersecting D-brane models, open string modes corresponding to matter fields and Higgs fields are localized at intersecting points between D-branes [16]. The flavor number is obtained as intersecting numbers of D-branes in conventional models.

Intersecting D-brane models have several interesting aspects from phenomenological viewpoints. In particular, their Yukawa couplings have a phenomenologically important feature. Calculation of Yukawa couplings in intersecting D-brane models [17–21] is similar to the Yukawa calculation of twisted strings in heterotic orbifold mod-

els [22–24][1]. These Yukawa couplings in both intersecting D-brane models and heterotic orbifold models depend on localized points of matter modes and moduli corresponding to the volume of compact space. Thus, Yukawa couplings are, in general, non-universal and can lead to suppressed values. Indeed, heterotic orbifold models have possibilities for realizing realistic Yukawa matrices [25]. Also intersecting D-brane models have the potential to lead to realistic Yukawa matrices, but most of the intersecting D-brane models, which have been constructed so far, seem to lead to the factorizable form of Yukawa matrices, $y_{ij} = a_i b_j$, that is, rank-one matrices. With this form of Yukawa matrices, one can derive only a non-vanishing mass for the third family, but vanishing values for other lighter masses and mixing angles [26]. Hence, it is quite important to study the new type of flavor structure which can lead to non-vanishing mixing angles and light fermion masses within the framework of intersecting D-brane models.

In [15] one of the authors has proposed an intersecting D-brane model with a new type of flavor structure, where quarks, leptons and Higgs doublets appear as composite fields. In this model, Yukawa couplings are dynamically generated. Since the origin of flavors is different from the one in the conventional models, the structure of Yukawa coupling matrices can be different from the one in conventional models. In this paper we investigate the structure of Yukawa coupling matrices of the model and show that the non-trivial form of Yukawa matrices is obtained in this framework of Yukawa coupling generations, e.g. non-vanishing mixing angles. Thus, our scenario is interesting if one wants to realize realistic values of fermion masses and mixing angles.

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[1] Yukawa couplings in heterotic models are calculable, because the string is solvable on orbifolds.

This paper is organized as follows. In Sect. 2, we give a brief review on intersecting D-brane models. Then, we discuss the structure of Yukawa coupling matrices in conventional models. We point out that it is difficult to have realistic structure in general. In Sect. 3 we briefly introduce a model of dynamical generation of Yukawa couplings which is proposed in [15]. In Sect. 4 a detailed analysis of the structure of Yukawa coupling matrices is given. We show that our framework of the Yukawa coupling generation derives a non-trivial structure of Yukawa matrices, which can be realistic. Section 5 is devoted to conclusions and discussions.

2 Intersecting D-brane models and Yukawa couplings

2.1 Intersecting D-brane models

Here we give a brief review on intersecting D-brane models. Consider the type IIA superstring theory compactified on $\mathbf{T}^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ orientifold, where $\mathbf{T}^6 = \mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$. The type IIA theory is invariant under the $\mathbf{Z}_2 \times \mathbf{Z}_2$ transformation

$$\theta : \quad X_\pm^k \to e^{\pm i 2\pi v_k} X_\pm^k, \tag{1}$$

$$\omega : \quad X_\pm^k \to e^{\pm i 2\pi w_k} X_\pm^k, \tag{2}$$

where $v = (0, 0, 1/2, -1/2, 0)$ and $w = (0, 0, 0, 1/2, -1/2)$ and

$$X_\pm^k = \begin{cases} \frac{1}{\sqrt{2}} \left(\pm X^{2k} + X^{2k+1} \right), & \text{for } k = 0, \\ \frac{1}{\sqrt{2}} \left(X^{2k} \pm i X^{2k+1} \right), & \text{for } k = 1, 2, 3, 4, \end{cases} \tag{3}$$

with space-time coordinates X^μ, $\mu = 0, 1, \ldots, 9$. The type IIA theory is also invariant under the ΩR transformation, where Ω is the world-sheet parity transformation and

$$R : \quad \begin{cases} X^i \to X^i, & \text{for } i = 0, 1, 2, 3, 4, 6, 8, \\ X^j \to -X^j, & \text{for } j = 5, 7, 9. \end{cases} \tag{4}$$

We mod out the theory by the action of θ, ω, ΩR and their independent combinations.

A D6$_a$-brane stretching over our three-dimensional space and winding in compact $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ space is specified by the winding numbers in each torus:

$$\left[\left(n_a^1, m_a^1 \right), \left(n_a^2, m_a^2 \right), \left(n_a^3, m_a^3 \right) \right]. \tag{5}$$

A D6$_a$-brane is always accompanied by its orientifold image D6$_{a'}$ whose winding numbers are

$$\left[\left(n_a^1, -m_a^1 \right), \left(n_a^2, -m_a^2 \right), \left(n_a^3, -m_a^3 \right) \right]. \tag{6}$$

The number of intersections between D6$_a$-brane and D6$_b$-brane is given by

$$I_{ab} = \prod_{i=1}^3 \left(n_a^i m_b^i - m_a^i n_b^i \right). \tag{7}$$

Table 1. General massless field contents on intersecting D6-branes. In the aa sector, the gauge symmetry is USp(N_a) or U($N_a/2$) corresponding to whether D6$_a$-brane is parallel or not to some O6-plane, respectively. In the $aa' + a'a$ sector, k is the number of tilted torus, and I_{aO6} is the sum of the intersection numbers between D6$_a$-brane and all O6-planes

sector	field
aa	U($N_a/2$) or USp(N_a) gauge multiplet.
	3 U($N_a/2$) adjoint or 3 USp(N_a) anti-symmetric tensor chiral multiplets.
$ab + ba$	I_{ab} ($\square_a, \overline{\square}_b$) chiral multiplets.
$ab' + b'a$	$I_{ab'}$ ($\square_a, \square_b$) chiral multiplets.
$aa' + a'a$	$\frac{1}{2} \left(I_{aa'} - \frac{4}{2^k} I_{aO6} \right)$ symmetric tensor chiral multiplets.
	$\frac{1}{2} \left(I_{aa'} + \frac{4}{2^k} I_{aO6} \right)$ anti-symmetric tensor chiral multiplets.

The intersecting angles, θ_a^i with $i = 1, 2, 3$, between D6$_a$-brane and X^4, X^6 and X^8 axes in each torus are given by

$$\theta_a^i = \tan^{-1} \left(\chi_i \frac{m_a^i}{n_a^i} \right), \tag{8}$$

where χ_i are the ratios of two radii of each torus: $\chi_i \equiv R_2^{(i)}/R_1^{(i)}$. The system has supersymmetry, if $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0$ is satisfied for all a. The configuration of intersecting D6-branes should satisfy the following Ramond–Ramond tadpole cancellation conditions,

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16, \tag{9}$$

$$\sum_a N_a n_a^1 m_a^2 m_a^3 = -16, \tag{10}$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 = -16, \tag{11}$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 = -16, \tag{12}$$

where N_a is the multiplicity of the D6$_a$-brane, and we are assuming three rectangular (untilted) tori. The Neveu–Schwarz–Neveu–Schwarz tadpoles are automatically cancelled, when Ramond–Ramond tadpole cancellation conditions and the supersymmetry conditions are satisfied.

There are four sectors of open string corresponding to on which D6-branes two ends of open string are fixed: aa, $ab + ba$, $ab' + b'a$, $aa' + a'a$ sectors. Each sector gives matter fields in four-dimensional low-energy effective theory. The general massless field contents are given in Table 1. A common problem of the model building in this framework is the appearance of the massless adjoint fields in the aa sector, since there are no massless matter fields in the adjoint representation under the standard model gauge group in Nature. These fields are expected to be massive in the case of a curved compact space, because these are the moduli fields of D6-brane configurations.

2.2 The Yukawa coupling structure in conventional models

The magnitude of Yukawa couplings among localized open string modes can be calculated by the conformal field theory technique like heterotic orbifold models. The important part of 3-point interactions y is evaluated by the classical part as [17–19]

$$y \sim e^{-S_{\mathrm{cl}}} \sim \prod_i e^{-\Sigma_i/2\pi\alpha'}. \tag{13}$$

Here, S_{cl} denotes the action of the classical string solution X_{cl}, which have the asymptotic behavior corresponding to local open string modes near intersecting points, and S_{cl} is obtained as a product of the triangle areas Σ_i on the ith T^2, which string sweeps to couple. These Yukawa couplings can lead to suppressed values when intersecting points are far away each other. Thus, intersecting D-brane models have the potential to realize the hierarchy of fermion masses and mixing angles.

Here, however, we discuss the difficulty to generate a realistic structure of Yukawa coupling matrices in the models from type IIA $T^6/(\mathrm{Z}_2 \times \mathrm{Z}_2)$ orientifolds with intersecting D-branes. The 6-dimensional torus is assumed to be factorizable: $T^6 = T^2 \times T^2 \times T^2$.

As an illustrating example, let us consider a toy model with three D6-branes, the D6$_3$-brane for $SU(3)_c$ gauge symmetry, the D6$_2$-brane for $SU(2)_{\mathrm{L}}$ gauge symmetry and the D6$_1$-brane for one component of $U(1)_Y$ gauge symmetry, and a very simple configuration of intersecting numbers,

$$I_{ab} = \begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 3 \\ -3 & -3 & 0 \end{pmatrix}_{ab}, \tag{14}$$

where $a, b = 1, 2, 3$. This system gives three generations of quarks, three left-handed quark doublets and three right-handed quark singlets, and one Higgs doublet with Yukawa couplings. The total intersecting number I_{ab} of D6$_a$-brane and D6$_b$-brane is given by the multiplications of three intersecting numbers of the D6$_a$-brane and D6$_b$-brane in each of three tori. The structure of the Yukawa coupling is not fully determined by the intersecting number, but it depends on the configuration of the intersection in each of three tori.

The intersecting number $I_{12} = 1$ means one Higgs doublet field localizing a point in each of three tori. The intersecting number $I_{13} = 3$ means three intersections in one of three tori and one intersection in the other two tori. The same is true for $I_{23} = 3$. Therefore, we have two different configurations of intersection:
(1) D6$_1$-brane and D6$_3$-brane intersect three times in a torus in which D6$_2$-brane and D6$_3$-brane also intersect three times, or
(2) D6$_1$-brane and D6$_3$-brane intersect three times in a torus in which D6$_2$-brane and D6$_3$-brane intersect once.

In both cases, the Yukawa couplings are evaluated by (13). The quantum part also contributes to Yukawa couplings, but gives $\mathcal{O}(1)$ of common factor. In case (1) the Yukawa matrix can become hierarchical but always diagonal. In this case the D6$_3$-brane should wind three times in a torus and intersect with each D6$_1$-brane and D6$_2$-brane once in each winding. This gives definitely three pairs of right-handed and left-handed quarks and defines three generations without mixing. The values of Yukawa couplings are determined by the areas of triangles which are determined by the place of each pair of quarks and Higgs doublet. In this case we cannot have flavor mixing. This structure of Yukawa coupling matrix seems unable to be modified by the quantum corrections in supersymmetric models.

In case (2) we always have the factorized structure of the Yukawa coupling matrix:

$$y_{ij} \propto a_i b_j, \tag{15}$$

where $i, j = 1, 2, 3$. In a torus where the D6$_1$-brane intersects three times with the D6$_3$-brane three left-handed quarks localize at one point and the Higgs doublet also localizes at one point. We have three triangle areas which determine the value of a_i or b_i. The same is true for the torus where the D6$_2$-brane intersects three times with the D6$_3$-brane. Therefore, the Yukawa coupling matrix is the multiplication of the contribution from each three torus. Furthermore, the Kähler metric depends on the twisted angles of the open string [20]. The twisted angle of open strings corresponding to three left-handed quarks is common, and we also have the same twisted angle for three right-handed quarks. Thus, the Kähler metric is relevant to only the overall magnitude of Yukawa matrices, but irrelevant to the ratios of entries in Yukawa matrices. In this case we cannot have a realistic mass spectrum, since the rank of the Yukawa coupling matrix is one. This structure of Yukawa coupling matrix also seems unable to be modified by the quantum corrections in supersymmetric models.

We have discussed the problem in the simplified model, but most of the models seem to have almost the same problem. In particular, it seems difficult to derive nonvanishing mixing angles as well as lighter fermion masses. There are several possibilities to overcome this difficulty.

First, we can try to construct models with some further complications by arranging the configuration of D6-branes. For example, introduction of many Higgs doublets may solve the problem [27], though it is not experimentally favored. To have three generations utilizing orientifold image D6-branes may give a way to solve the problem, though we will have relatively many unwanted exotic fields.

The second way is to change the structure of the compactified space. Leaving from the factorizable case $T^6 = T^2 \times T^2 \times T^2$ to $T^6 = T^4 \times T^2$, for example, may make the situation completely change. But the physics becomes less intuitive and the analysis becomes much more complicated.

The third way is to change the origin of the generation. This is the way we are going to take in this paper. In the next section, we introduce a model in which the generation is not originated from the multiple intersection of D6-branes.

3 A model of dynamically generated Yukawa couplings

In this section we give a brief description of the model which has been introduced in [15].

Table 2. Configuration of intersecting D6-branes. All three tori are considered to be rectangular (untilted). Three D6-branes, $D6_4$, $D6_5$ and $D6_6$, are on top of some O6-planes

D6-brane	winding number	multiplicity
$D6_1$	$[(1,-1),(1,1),(1,0)]$	4
$D6_2$	$[(1,1),(1,0),(1,-1)]$	$6+2$
$D6_3$	$[(1,0),(1,-1),(1,1)]$	$2+2$
$D6_4$	$[(1,0),(0,1),(0,-1)]$	12
$D6_5$	$[(0,1),(1,0),(0,-1)]$	8
$D6_6$	$[(0,1),(0,-1),(1,0)]$	12

The D6-brane configuration of the model is given in Table 2. Both tadpole cancellation conditions and supersymmetry conditions are satisfied in this configuration under the conditions of $\chi_1 = \chi_2 = \chi_3 \equiv \chi$. The $D6_2$-brane system consists of two parallel D6-branes with multiplicities six and two which are separated in the second torus in a consistent way with the orientifold projections. The $D6_3$-brane system consists of two parallel D6-branes with multiplicity two which are separated in the first torus in a consistent way to the orientifold projections. The $D6_1$-, $D6_2$- and $D6_3$-branes give the gauge symmetries of $U(2)_L = SU(2)_L \times U(1)_L$, $U(3)_c \times U(1) = SU(3)_c \times U(1)_c \times U(1)$ and $U(1)_1 \times U(1)_2$, respectively. The hypercharge is defined as

$$\frac{Y}{2} = \frac{1}{2}\left(\frac{Q_c}{3} - Q\right) + \frac{1}{2}(Q_1 - Q_2), \qquad (16)$$

where Q_c, Q, Q_1 and Q_2 are charges of $U(1)_c$, $U(1)$, $U(1)_1$ and $U(1)_2$, respectively. The additional non-anomalous $U(1)$ charge, Q_R, is defined as
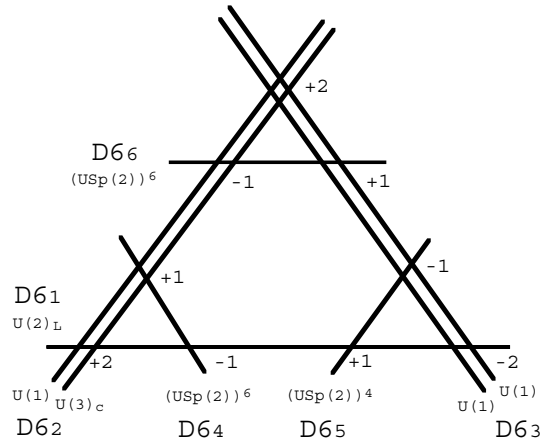
$$Q_R = Q_1 - Q_2. \qquad (17)$$

The remaining three $U(1)$ gauge symmetries which are generated by Q_L (namely $U(1)_L$), $Q_c + Q$ and $Q_1 + Q_2$ are anomalous, and their gauge bosons have masses of the order of the string scale. These three anomalous $U(1)$ gauge symmetries are independent from the two non-anomalous $U(1)$ gauge symmetries: $\text{tr}((Y/2)Q_L) = 0$, for example.

A schematic picture of the configuration of intersecting D6-branes of this model is given in Fig. 1. There are no $ab' + b'a$, $aa' + a'a$ sectors of open string in this configuration. The massless particle contents are given in Table 3. In this table it is assumed that all twelve D6-branes of $D6_4$ are on top of one of eight O6-branes with the same winding numbers. The same is also assumed for eight and twelve D6-branes of $D6_5$ and $D6_6$.

We break three $USp(12)_{D6_4}$, $USp(8)_{D6_5}$ and $USp(12)_{D6_6}$ gauge symmetries to the factors of $USp(2)$ gauge symmetries by configuring D6-branes of $D6_4$, $D6_5$ and $D6_6$ as in Fig. 2. The resultant gauge symmetries are respectively as follows:

$$USp(12)_{D6_4} \longrightarrow \overset{6}{\underset{\alpha=1}{\otimes}} USp(2)_{D6_4,\alpha}, \qquad (18)$$

$$USp(8)_{D6_5} \longrightarrow \overset{4}{\underset{a=1}{\otimes}} USp(2)_{D6_5,a}, \qquad (19)$$

Fig. 1. Schematic picture of the configuration of intersecting D6-branes. This picture describes only the configuration of the intersection of the D6-branes with each other, and the relative place of each D6-brane has no meaning. The number at the intersection point between $D6_a$- and $D6_b$-branes denotes the intersection number I_{ab} with $a < b$

Table 3. Low-energy particle contents before "hypercolor" confinement. The fields from the aa sectors are neglected for simplicity

sector	$SU(3)_c \times SU(2)_L \times USp(8)$ $\times USp(12)_{D6_4} \times USp(12)_{D6_6}$ $(Y/2, Q_R)(Q_L, Q_c+Q, Q_1+Q_2)$	field
$D6_1 \cdot D6_2$	$(3^*, 2, 1, 1, 1)_{(-1/6,0)(+1,-1,0)} \times 2$	$\bar{q}_i$
	$(1, 2, 1, 1, 1)_{(+1/2,0)(+1,-1,0)} \times 2$	$\bar{l}_i$
$D6_1 \cdot D6_4$	$(1, 2, 1, 12, 1)_{(0,0)(-1,0,0)}$	D
$D6_2 \cdot D6_4$	$(3, 1, 1, 12, 1)_{(+1/6,0)(0,+1,0)}$	C
	$(1, 1, 1, 12, 1)_{(-1/2,0)(0,+1,0)}$	N
$D6_1 \cdot D6_3$	$(1, 2, 1, 1, 1)_{(+1/2,+1)(-1,0,+1)} \times 2$	$H_i^{(1)}$
	$(1, 2, 1, 1, 1)_{(-1/2,-1)(-1,0,+1)} \times 2$	$\bar{H}_i^{(2)}$
$D6_1 \cdot D6_5$	$(1, 2, 8, 1, 1)_{(0,0)(+1,0,0)}$	T
$D6_3 \cdot D6_5$	$(1, 1, 8, 1, 1)_{(+1/2,+1)(0,0,-1)}$	$T^{(+)}$
	$(1, 1, 8, 1, 1)_{(-1/2,-1)(0,0,-1)}$	$T^{(-)}$
$D6_2 \cdot D6_3$	$(3, 1, 1, 1, 1)_{(-1/3,-1)(0,+1,-1)} \times 2$	$\bar{d}_i$
	$(3, 1, 1, 1, 1)_{(+2/3,+1)(0,+1,-1)} \times 2$	$\bar{u}_i$
	$(1, 1, 1, 1, 1)_{(-1,-1)(0,+1,-1)} \times 2$	$\bar{e}_i$
	$(1, 1, 1, 1, 1)_{(0,+1)(0,+1,-1)} \times 2$	$\bar{\nu}_i$
$D6_2 \cdot D6_6$	$(3^*, 1, 1, 1, 12)_{(-1/6,0)(0,-1,0)}$	$\bar{C}$
	$(1, 1, 1, 1, 12)_{(+1/2,0)(0,-1,0)}$	$\bar{N}$
$D6_3 \cdot D6_6$	$(1, 1, 1, 1, 12)_{(+1/2,+1)(0,0,+1)}$	$\bar{D}^{(+)}$
	$(1, 1, 1, 1, 12)_{(-1/2,-1)(0,0,+1)}$	$\bar{D}^{(-)}$

$$USp(12)_{D6_6} \longrightarrow \overset{6}{\underset{\alpha=1}{\otimes}} USp(2)_{D6_6,\alpha}. \qquad (20)$$

All of these $USp(2)$ gauge intersections can be naturally stronger than any other unitary gauge interactions. If we choose $\kappa_4 M_s \sim 1$ and $\chi \sim 0.1$, where $\kappa_4 = \sqrt{8\pi G_N}$, $M_s = 1/\sqrt{\alpha'}$, the dynamical scales of all $USp(2)$ gauge interactions are of the order of M_s, and the values of the standard model gauge coupling constants are reasonably

D6$_4$

D6$_5$

D6$_6$
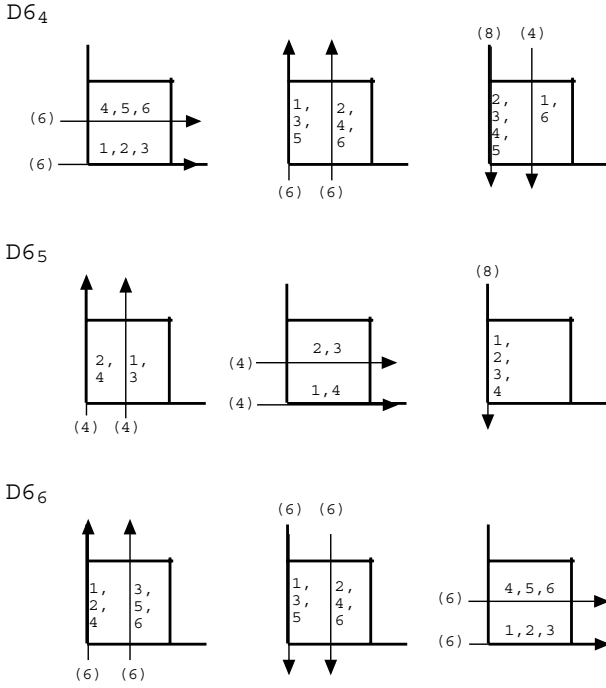
Fig. 2. Configurations of twelve, eight and twelve D6-branes of D6$_4$-, D6$_5$- and D6$_6$-branes, respectively. The numbers in brackets are multiplicities of D6-brane stacks, and the numbers without brackets specify one of the USp(2) gauge groups in (18), (19) and (20). See the footnote for details

Table 4. Field contents of the left-handed sector. Here, $i = 1, 2$ and $\alpha = 1, 2, \ldots, 6$

sector	$(\mathrm{SU}_{(3)_c} \times \mathrm{SU}_{(2)_\mathrm{L}}) \times (\mathrm{USp}(2)_1 \times \mathrm{USp}(2)_2 \times$	field
	$\mathrm{USp}(2)_3 \times \mathrm{USp}(2)_4 \times \mathrm{USp}(2)_5 \times \mathrm{USp}(2)_6)_{D6_4}$	
	$(Y/2, Q_\mathrm{R})(Q_\mathrm{L}, Q_c + Q, Q_1 + Q_2)$	
$D6_1 \cdot D6_2$	$(3^*, 2)(1,1,1,1,1,1)_{(-1/6,0)(+1,-1,0)} \times 2$	$\bar{q}_i$
	$(1, 2)(1,1,1,1,1,1)_{(+1/2,0)(+1,-1,0)} \times 2$	$\bar{l}_i$
$D6_1 \cdot D6_4$	$(1, 2)(2,1,1,1,1,1)_{(0,0)(-1,0,0)}$	D_α
	$(1, 2)(1,2,1,1,1,1)_{(0,0)(-1,0,0)}$	
	$(1, 2)(1,1,2,1,1,1)_{(0,0)(-1,0,0)}$	
	$(1, 2)(1,1,1,2,1,1)_{(0,0)(-1,0,0)}$	
	$(1, 2)(1,1,1,1,2,1)_{(0,0)(-1,0,0)}$	
	$(1, 2)(1,1,1,1,1,2)_{(0,0)(-1,0,0)}$	
$D6_2 \cdot D6_4$	$(3, 1)(2,1,1,1,1,1)_{(+1/6,0)(0,+1,0)}$	C_α
	$(3, 1)(1,2,1,1,1,1)_{(+1/6,0)(0,+1,0)}$	
	$(3, 1)(1,1,2,1,1,1)_{(+1/6,0)(0,+1,0)}$	
	$(3, 1)(1,1,1,2,1,1)_{(+1/6,0)(0,+1,0)}$	
	$(3, 1)(1,1,1,1,2,1)_{(+1/6,0)(0,+1,0)}$	
	$(3, 1)(1,1,1,1,1,2)_{(+1/6,0)(0,+1,0)}$	
	$(1, 1)(2,1,1,1,1,1)_{(-1/2,0)(0,+1,0)}$	N_α
	$(1, 1)(1,2,1,1,1,1)_{(-1/2,0)(0,+1,0)}$	
	$(1, 1)(1,1,2,1,1,1)_{(-1/2,0)(0,+1,0)}$	
	$(1, 1)(1,1,1,2,1,1)_{(-1/2,0)(0,+1,0)}$	
	$(1, 1)(1,1,1,1,2,1)_{(-1/2,0)(0,+1,0)}$	
	$(1, 1)(1,1,1,1,1,2)_{(-1/2,0)(0,+1,0)}$	

of $\mathcal{O}(0.01)$ at the string scale. We call these strong USp(2) interactions "hypercolor" interactions.[2]

The field content in the intersecting D6$_1$–D6$_2$–D6$_4$ sector (left-handed sector) is given in Table 4. The confinement of six USp(2)$_{D6_4,\alpha}$ gauge interactions gives six generations of left-handed quark and lepton doublets:

$$C_\alpha D_\alpha \sim q_\alpha, \qquad N_\alpha D_\alpha \sim l_\alpha, \qquad (21)$$

where $\alpha = 1, 2, \ldots, 6$. Two of these six left-handed quark doublets and two of these six left-handed lepton doublets become massive through the string-level Yukawa interactions of

$$W_{\mathrm{left}} = \sum_{i,\alpha} g_{i\alpha}^{\mathrm{left}-q} \bar{q}_i C_\alpha D_\alpha + \sum_{i,\alpha} g_{i\alpha}^{\mathrm{left}-l} \bar{l}_i N_\alpha D_\alpha, \quad (22)$$

where $i = 1, 2$. We have four massless generations of left-handed quark and lepton doublets. The values the of masses

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[2] We have many vector-like matters in the configuration of Fig. 2, since, for example, D6-branes of D6$_4$ and D6$_5$ overlap in the third torus and intersect in other tori. These vector-like matters spoil asymptotic freedom of "hypercolor" dynamics. But, these vector-like matters get masses of Planck scale, if we move four D-branes of each USp(2) (two D-branes and their images) a little away from orientifold planes in one torus in a consistent way with orientifold projections and avoid overlapping. This procedure does not change the gauge symmetry and chiral matter contents of the model. In the estimation of the Yukawa couplings, we do not consider this fine structure of the D-brane configuration, since it does not change our main results.

are given as $g_{i\alpha}^{\mathrm{left}-q} \Lambda_\mathrm{L}$ and $g_{i\alpha}^{\mathrm{left}-l} \Lambda_\mathrm{L}$, where Λ_L denotes the dynamical scale of USp(2)$_{D6_4,\alpha}$. The detailed structure of the Yukawa coupling matrix $g_{i\alpha}^{\mathrm{left}-q}$ is discussed in the next section. Exactly the same happens in the intersecting D6$_2$–D6$_3$–D6$_6$ sector (right-handed sector). The confinement of six USp(2)$_{D6_6,\alpha}$ gauge interactions gives six generations of right-handed quarks and leptons:

$$\bar{C}_\alpha \bar{D}_\alpha^{(-)} \sim u_\alpha, \qquad \bar{C}_\alpha \bar{D}_\alpha^{(+)} \sim d_\alpha,$$
$$\bar{N}_\alpha \bar{D}_\alpha^{(-)} \sim \nu_\alpha, \qquad \bar{N}_\alpha \bar{D}_\alpha^{(+)} \sim e_\alpha. \qquad (23)$$

We have four massless generations of right-handed quarks and leptons through the string-level Yukawa interactions of

$$W_{\mathrm{right}} = \sum_{i,\alpha} g_{i\alpha}^{\mathrm{right}-u} \bar{u}_i \bar{C} \bar{D}^{(-)} + \sum_{i,\alpha} g_{i\alpha}^{\mathrm{right}-d} \bar{d}_i \bar{C} \bar{D}^{(+)}$$
$$+ \sum_{i,\alpha} g_{i\alpha}^{\mathrm{right}-\nu} \bar{\nu}_i \bar{N} \bar{D}^{(-)} + \sum_{i,\alpha} g_{i\alpha}^{\mathrm{right}-e} \bar{e}_i \bar{N} \bar{D}^{(+)}. \qquad (24)$$

The detailed structure of the Yukawa coupling matrices $g_{i\alpha}^{\mathrm{right}-u}$ and $g_{i\alpha}^{\mathrm{right}-d}$ is also discussed in the next section. The values of masses are given as $g_{i\alpha}^{\mathrm{right}-u} \Lambda_\mathrm{R}$ and $g_{i\alpha}^{\mathrm{right}-d} \Lambda_\mathrm{R}$ with the dynamical scale of USp(2)$_{D6_6,\alpha}$. Here, we stress that the origin of generation in this model is not the multiple intersections of D6-branes, but the number of different D6-branes with the same winding numbers.

A similar thing happens in the intersecting D6$_1$–D6$_3$– D6$_5$ sector (Higgs sector). The confinement of two USp(2)$_{\text{D6}_5,a}$ gauge interactions gives eight composite Higgs fields:

$$T_a T^{(+)} \sim H_a^{(2)}, \qquad T_a T^{(-)} \sim \bar{H}_a^{(1)}, \tag{25}$$

where $a = 1, 2, 3, 4$. Four of these eight composite Higgs fields become massive through the string-level Yukawa interactions of

$$W_{\text{Higgs}} = \sum_{i,a} g_{ia}^{(1)} H_i^{(1)} T_a T_a^{(-)} + \sum_{i,a} g_{ia}^{(2)} \bar{H}_i^{(2)} T_a T_a^{(+)}. \tag{26}$$

The masses of four pairs of Higgs fields are given as $g_{ia}^{(1)} \Lambda_H$ and $g_{ia}^{(2)} \Lambda_H$, where Λ_H is the scale of dynamics of USp(2)$_{\text{D6}_5,a}$. Two pairs of composite Higgs fields remain massless.

From Fig. 1 we see the existence of the following interactions:

$$
\begin{aligned}
W_{6-\text{fields}} = & \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}^u}{M_s^3} [C_\alpha D_\alpha][\bar{C}_\beta \bar{D}_\beta^{(-)}][T_a T_a^{(+)}] \\
& + \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}^d}{M_s^3} [C_\alpha D_\alpha][\bar{C}_\beta \bar{D}_\beta^{(+)}][T_a T_a^{(-)}] \\
& + \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}^\nu}{M_s^3} [N_\alpha D_\alpha][\bar{N}_\beta \bar{D}_\beta^{(-)}][T_a T_a^{(+)}] \\
& + \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g_{\alpha\beta a}^e}{M_s^3} [N_\alpha D_\alpha][\bar{N}_\beta \bar{D}_\beta^{(+)}][T_a T_a^{(-)}].
\end{aligned}
\tag{27}
$$

After the "hypercolor" confinement these interactions give Yukawa interactions for the mass generation of quarks and leptons. The Yukawa coupling matrices are given by

$$y_{\alpha\beta a}^u \simeq g_{\alpha\beta a}^u \frac{\Lambda_\text{L} \Lambda_\text{R} \Lambda_H}{M_s^3} \sim g_{\alpha\beta a}^u, \tag{28}$$

$$y_{\alpha\beta a}^d \simeq g_{\alpha\beta a}^d \frac{\Lambda_\text{L} \Lambda_\text{R} \Lambda_H}{M_s^3} \sim g_{\alpha\beta a}^d, \tag{29}$$

$$y_{\alpha\beta a}^\nu \simeq g_{\alpha\beta a}^\nu \frac{\Lambda_\text{L} \Lambda_\text{R} \Lambda_H}{M_s^3} \sim g_{\alpha\beta a}^\nu, \tag{30}$$

$$y_{\alpha\beta a}^e \simeq g_{\alpha\beta a}^e \frac{\Lambda_\text{L} \Lambda_\text{R} \Lambda_H}{M_s^3} \sim g_{\alpha\beta a}^e, \tag{31}$$

since all the scales of dynamics, Λ_L, Λ_R and Λ_H are of the order of the string scale M_s. The detailed structure of these Yukawa coupling matrices $g_{\alpha\beta a}^u$ and $g_{\alpha\beta a}^d$ is investigated in the next section.

4 The structure of Yukawa coupling matrices

Here we investigate the structure of Yukawa coupling matrices for quark masses of the model introduced in the previous section. We do not require the realistic μ-term (realistic value of the Higgs mass), since the aim of the analysis is simply to show the possibility of having a non-trivial structure of Yukawa coupling matrices, e.g. non-vanishing mixing angles. The possibility of entire mass generation of quarks and leptons in this kind of models will be investigated in future works.

First, we evaluate the Yukawa coupling matrices $g_{i\alpha}^{\text{left}-q}$ in (22) and $g_{i\alpha}^{\text{right}-u}$ and $g_{i\alpha}^{\text{right}-d}$ in (24) by using (13). Three Yukawa-interacting fields are localized at three intersection points of these three D6-branes. The Yukawa couplings, $g_{i\alpha}^{\text{left}-q}$, $g_{i\alpha}^{\text{right}-u}$ and $g_{i\alpha}^{\text{right}-d}$, are obtained as

$$g^{\text{left}-q} = \begin{pmatrix} \varepsilon_3 & \varepsilon_2 & 1 & \varepsilon_1^2 \varepsilon_2 & \varepsilon_1^2 & \varepsilon_1^2 \varepsilon_2 \varepsilon_3 \\ \varepsilon_1^2 \varepsilon_3 & \varepsilon_1^2 \varepsilon_2 & \varepsilon_1^2 & \varepsilon_2 & 1 & \varepsilon_2 \varepsilon_3 \end{pmatrix}, \tag{32}$$

$$g^{\text{right}-u} = \begin{pmatrix} 1 & \varepsilon_2 & \varepsilon_1 & \varepsilon_2 \varepsilon_3^2 & \varepsilon_1 \varepsilon_3^2 & \varepsilon_1 \varepsilon_2 \varepsilon_3^2 \\ \varepsilon_3^2 & \varepsilon_2 \varepsilon_3^2 & \varepsilon_1 \varepsilon_3^2 & \varepsilon_2 & \varepsilon_1 & \varepsilon_1 \varepsilon_2 \end{pmatrix}, \tag{33}$$

$$g^{\text{right}-d} = \begin{pmatrix} \varepsilon_1 & \varepsilon_1 \varepsilon_2 & 1 & \varepsilon_1 \varepsilon_2 \varepsilon_3^2 & \varepsilon_3^2 & \varepsilon_2 \varepsilon_3^2 \\ \varepsilon_1 \varepsilon_3^2 & \varepsilon_1 \varepsilon_2 \varepsilon_3^2 & \varepsilon_3^2 & \varepsilon_1 \varepsilon_2 & 1 & \varepsilon_2 \end{pmatrix}, \tag{34}$$

where $\varepsilon_i = \exp(-A_i/2\pi\alpha')$ and A_i is the 1/8 of the area of the ith torus. In general, classical solutions with larger area also contribute to the Yukawa couplings. However, we have used the approximation that the classical action corresponding to the minimum area contributes dominantly to the Yukawa couplings. In the following we assume that all A_i are larger than $2\pi\alpha'$, namely $\varepsilon < 1$. From these results we can say in good approximation that the massless left-handed quark doublets are q_1, q_2, q_4 and q_6, and the massless right-handed down-type quarks are d_1, d_2, d_4 and d_6. For right-handed up-type quarks, u_2, u_3, u_6 and one linear combination of u_4 and u_5 are approximately massless. Especially, in case of $\varepsilon_1 \ll \varepsilon_2$, u_2, u_3, u_5 and u_6 are approximately massless, and in case of $\varepsilon_2 \ll \varepsilon_1$, u_2, u_3, u_4 and u_6 are approximately massless.

Although it is not easy to calculate the 6-point coupling matrices of $g_{\alpha\beta a}^u$ and $g_{\alpha\beta a}^d$ in (27), generic n-point functions have been analyzed in [28]. Our purpose is not to realize precisely realistic fermion masses and mixing angles, but to show that a new type of non-trivial Yukawa matrices can be derived in our scenario, e.g. suppressed but non-vanishing mixing angles. For such purpose, an approximation, which is reliable to order estimation on the suppression factors, is sufficient. The 6-point function consists of classical and quantum contributions, i.e. $Z_{\text{cl}} Z_q$, like the 3-point and 4-point functions. The classical part Z_{cl} is important for our purpose because that can lead to a suppression factor, while the quantum part is expected to be of O(1) and does not contribute to lead to a suppression factor. The classical part is written as $Z_{\text{cl}} = \sum_{X_{\text{cl}}} e^{-S_{\text{cl}}}$ like (13), where X_{cl} are the classical string solutions, which have the asymptotic behavior corresponding to local open string modes near intersecting points. the minimum The minimum action corresponds to the sum of the areas of hexagons with six intersecting points on each torus [28]. Hence, in order to give an order estimation of the 6-point couplings $g_{\alpha\beta a}^{u,d}$, we use the approximation that $g_{\alpha\beta a}^{u,d} \sim \prod_i e^{-\Sigma_i/2\pi\alpha'}$, which is

the same as (13) except for simply replacing the triangular areas Σ_i by the hexagonal areas. This approximation gives a sufficient order estimation of the exponential suppression, enough for our purpose, though some sub-leading corrections are neglected. Actually, in our model, all the hexagons reduce to triangles, because some intersecting points always coincide, and only three of the six sit at the different places. Hence, we obtain the following result:

$$g^u_{\alpha\beta a=1} = g^d_{\alpha\beta a=1} =$$

$$
\begin{pmatrix}
\varepsilon_1\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_3 & \varepsilon_1^2\varepsilon_2\varepsilon_3 \\
\varepsilon_1\varepsilon_2 & \varepsilon_1\varepsilon_2^2 & \varepsilon_1^2\varepsilon_2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3^2 \\
\varepsilon_1 & \varepsilon_1\varepsilon_2 & \varepsilon_1^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1\varepsilon_2 & \varepsilon_1\varepsilon_2^2 & \varepsilon_2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 & \varepsilon_2\varepsilon_3^2 & \varepsilon_2^2\varepsilon_3^2 \\
\varepsilon_1 & \varepsilon_1\varepsilon_2 & 1 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_3^2 & \varepsilon_2\varepsilon_3^2 \\
\varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_2^2\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_2^2\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_2^2\varepsilon_3
\end{pmatrix}_{\alpha\beta} , \qquad (35)
$$

$$g^u_{\alpha\beta a=2} = g^d_{\alpha\beta a=2} =$$

$$
\begin{pmatrix}
\varepsilon_2^2\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_2^2\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_2^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 \\
\varepsilon_2 & 1 & \varepsilon_1\varepsilon_2 & \varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_3^2 \\
\varepsilon_2^2 & \varepsilon_2 & \varepsilon_1\varepsilon_2^2 & \varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1^2\varepsilon_2 & \varepsilon_1^2 & \varepsilon_1\varepsilon_2 & \varepsilon_1^2\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_3^2 \\
\varepsilon_1^2\varepsilon_2^2 & \varepsilon_1^2\varepsilon_2 & \varepsilon_1\varepsilon_2^2 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1^2\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_3
\end{pmatrix}_{\alpha\beta} , \qquad (36)
$$

$$g^u_{\alpha\beta a=3} = g^d_{\alpha\beta a=3} =$$

$$
\begin{pmatrix}
\varepsilon_1\varepsilon_2^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3 & \varepsilon_1^2\varepsilon_2\varepsilon_3 \\
\varepsilon_1\varepsilon_2 & \varepsilon_1 & \varepsilon_1^2\varepsilon_2 & \varepsilon_1\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_3^2 \\
\varepsilon_1\varepsilon_2^2 & \varepsilon_1\varepsilon_2 & \varepsilon_1^2\varepsilon_2^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3^2 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1\varepsilon_2 & \varepsilon_1 & \varepsilon_2 & \varepsilon_1\varepsilon_3^2 & \varepsilon_2\varepsilon_3^2 & \varepsilon_3^2 \\
\varepsilon_1\varepsilon_2^2 & \varepsilon_1\varepsilon_2 & \varepsilon_2^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_2^2\varepsilon_3^2 & \varepsilon_2\varepsilon_3^2 \\
\varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_3
\end{pmatrix}_{\alpha\beta} , \qquad (37)
$$

$$g^u_{\alpha\beta a=4} = g^d_{\alpha\beta a=4} =$$

$$
\begin{pmatrix}
\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_3 & \varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 \\
\varepsilon_2 & \varepsilon_2^2 & \varepsilon_1\varepsilon_2 & \varepsilon_2^2\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 \\
1 & \varepsilon_2 & \varepsilon_1 & \varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1^2\varepsilon_2 & \varepsilon_1^2\varepsilon_2^2 & \varepsilon_1\varepsilon_2 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_2^2\varepsilon_3^2 \\
\varepsilon_1^2 & \varepsilon_1^2\varepsilon_2 & \varepsilon_1 & \varepsilon_1^2\varepsilon_2\varepsilon_3^2 & \varepsilon_1\varepsilon_3^2 & \varepsilon_1\varepsilon_2\varepsilon_3^2 \\
\varepsilon_1^2\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1^2\varepsilon_2^2\varepsilon_3 & \varepsilon_1\varepsilon_2\varepsilon_3 & \varepsilon_1\varepsilon_2^2\varepsilon_3
\end{pmatrix}_{\alpha\beta} . \qquad (38)
$$

Since we have six pair of Higgs fields and only the composite Higgs fields have Yukawa couplings, there are eight Yukawa coupling matrices for quark masses.

All of these Yukawa matrices are non-trivial in the sense that they differ from the factorizable form of (15). Detailed calculation is necessary for full analysis of these Yukawa matrices by varying parameters. However, our purpose is not to analyze this model in detail, but to show the possibility for leading to non-trivial results in our scenario, i.e.

non-vanishing mass ratios and mixing angles, which can be realistic. Therefore, here we show a simple case.

Now, let us consider the case with $\varepsilon_2 \ll \varepsilon_1$. In this case, the modes u_1 and u_5 as well as q_3, q_5, d_3 and d_5 dominantly gain large masses through the interactions of (22) and (24) with the coupling matrices of (32), (33) and (34). The other modes approximately correspond to four generations. For example, let us consider the Yukawa matrices, $g^u_{a=2}$ and $g^d_{a=3}$, that is, we assume that one pair of composite Higgs doublet fields, $(\bar{H}_3^{(1)}, H_2^{(2)})$, is dominant to generate quark mass matrices at the weak scale. Then we consider the parameter region $\varepsilon_2 \ll \varepsilon_3 \ll \varepsilon_1(\sim 1)$, and in this parameter region the matter fields (q_2, q_4, q_6), (u_2, u_6) and (d_2, d_6) have large entries in the Yukawa matrices $g^u_{a=2}$ and $g^d_{a=3}$. Here, let us concentrate ourselves on two heavy modes among four generations. The (2×2) submatrices with large entries in the Yukawa matrices $g^u_{a=2}$ and $g^d_{a=3}$ are written up to $O(\varepsilon_3^2)$ as

$$g^u_{a=2} \to \begin{pmatrix} 1 & 0 \\ \varepsilon_1^2\varepsilon_3 & \varepsilon_1\varepsilon_3 \end{pmatrix} , \qquad (39)$$

for the up sector,

$$g^d_{a=3} \to \begin{pmatrix} \varepsilon_1 & 0 \\ \varepsilon_1\varepsilon_3 & \varepsilon_3 \end{pmatrix} , \qquad (40)$$

for the down sector in the field basis $((q_2 + q_4)/\sqrt{2}, q_6)$, (u_2, u_6), (d_2, d_6).

Now we can calculate mass eigenvalues of two heavy modes among four generations, i.e. $m_{u,3}$ and $m_{u,4}$ for the up sector and $m_{d,3}$ and $m_{d,4}$ for the down sector, and their mixing angle V_{34}. The mass ratios and the mixing angle is obtained as

$$\frac{m_{u,3}}{m_{u,4}} \approx \varepsilon_1\varepsilon_3, \qquad \frac{m_{d,3}}{m_{d,4}} \approx \frac{\varepsilon_3}{\varepsilon_1}, \qquad V_{34} \approx \varepsilon_3, \qquad (41)$$

that is, we have the following relation:

$$\sqrt{\frac{m_{u,3}}{m_{u,4}}\frac{m_{d,3}}{m_{d,4}}} \approx V_{34}, \qquad (42)$$

at the composite scale.

It is interesting to compare these results with the experimental values of quark mass ratios, $\frac{m_c}{m_t}$ and $\frac{m_s}{m_b}$, and the mixing angle V_{cb}. At the weak scale, the experimental values of mass ratios, $\frac{m_c}{m_t} = 0.0038$ and $\frac{m_s}{m_b} = 0.025$, lead to

$$\sqrt{\frac{m_c}{m_t}\frac{m_s}{m_b}} = 0.01, \qquad (43)$$

and the mixing angle is

$$V_{cb} = 0.04. \qquad (44)$$

We find that the values of parameters $\varepsilon_1 \sim 0.5$ and $\varepsilon_3 \sim 0.01$ lead to an almost realistic structure of the quark Yukawa coupling matrices. Note that our approximation is reliable as order estimation.

We have shown that the structure of the Yukawa coupling matrices at least for the (2×2) sub-matrices can be realistic in the simple case through our scenario of the dynamical generation of Yukawa coupling matrices. The most relevant fact is the different origin of the generation from that in the conventional intersecting D-brane models. The origin of the generation is not the multiple intersection of D-branes, but many different D-branes with the same multiplicity and the same winding numbers.

We give several comments in order before closing this section.

In this model a large Yukawa coupling is obtained in case of which all six localization points of "preons" are coincides in all three tori. Therefore, it seems very difficult to obtain the Yukawa coupling of the order of unity. But we have shown that it is possible and can accidentally happen (see (39)).

For more rigorous investigations, we should take care of the normalization of fields. The low-energy effective fields in string theory should be normalized by considering the moduli dependence of the Kähler potential. A concrete analysis on the moduli dependence of the Kähler potential in type IIA intersecting D-brane models is given in [20]. The normalization of the low-energy effective field after "hypercolor" confinement should also be considered more precisely. Unfortunately, there is no established method to obtain the Kähler potentials for composite fields in strong coupling gauge theories in four dimensions, except for some special cases [29, 30].

5 Conclusions

The structure of Yukawa coupling matrices in models from type IIA $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ orientifolds ($T^6 = T^2 \times T^2 \times T^2$) with intersecting D-branes has been investigated. There is a difficulty to have a realistic Yukawa coupling matrices in the models in which the generation structure of quarks and leptons is originated from the multiple intersection of D-branes. On the other hand, it has been shown that the structure of Yukawa coupling matrices in the models with dynamical generation of Yukawa coupling matrices is non-trivial. Indeed, realistic values of the mixing angles V_{cb} and mass ratios m_c/m_t and m_s/m_b can be realized. The most relevant fact is the different origin of the generation. The origin of the generation is not the multiple intersection of D-branes, but many different D-branes with the same multiplicity and the same winding numbers.

Here we give a comment on the origin of CP violating phases. Inclusion of Wilson lines is one source of CP phases in Yukawa matrices [17]. As another source, the holomorphic dynamical scales $\Lambda_{R,L,H}$ may provide CP phases in effective Yukawa couplings in this class of models.

Although there is the possibility of having realistic Yukawa coupling matrices, this class of models with dynamical generation of Yukawa coupling matrices have many other phenomenological issues to be solved: constructing models with three generations and light Higgs pairs with a realistic μ-term for electroweak symmetry breaking, realization of the supersymmetry breaking, and so on. In addition to these which are common in all the supersymmetric type IIA intersecting D-brane models, composite models usually have the problem of rapid proton decay. It would be very interesting to explore more realistic models by incorporating the results of efforts of model building based on the field theory into the framework of the intersecting D-brane model.

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